UNIT - IV

GRAVITATION

Newton's law of gravitation

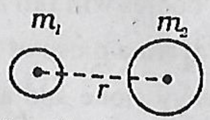
Newton suggested that gravitational force might very inversely as the square of the distance between the bodies. He realized that this force of attraction was a case of universal attraction between any two bodies present anywhere in the universe and proposed universal gravitational law.

The law states that every particle of matter in the universe attracts every other particle with a force with is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

Consider two bodies of masses m_1 and m_2 with their centres separated by a distance r. The gravitational force between them is

$$F \propto m_1 m_2$$

$$F \propto 1/r^2$$



Gravitational force

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant.

If $m_1 = m_2 = 1$kg and r = 1 m, then F = G.

Hence, the Gravitational constant 'G' is numerically equal to the gravitational force of attraction between two bodies of mass 1 kg each separated by a distance of 1 m. The value of G is $6.67 \times 10^{-11} Nm^2kg^{-2}$ and its dimensional formula is $M^{-1}L^3T^{-2}$.

Special features of the law

1. The gravitational force between two bodies is an action and reaction pair.
2. The gravitational force is very small in the case of lighter bodies. It is appreciable in the case of massive bodies. The gravitational force between the Sun and the Earth is of the order of $10^{27}N$.

Acceleration due to gravity

Galileo was the first to make a systematic study of the motion of a body under the gravity of the Earth. He dropped various objects from the leaning tower of Pisa and made analysis of their motion under gravity. He came to the conclusion that "in the absence of air. all bodies will fall at the same rate". It is the air resistance that slows down a piece of paper or a parachute falling under gravity. If a heavy stone and a parachute are dropped where there is no air, both will fall together at the same rate.

Variation of acceleration due to gravity

(i) Variation of g with altitude

Let P be a point on the surface of the Earth and Q be a point at an altitude h. Let the mass of the Earth be M and radius of the Earth be R. Consider the Earth as a spherical shaped body.

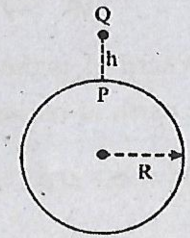
The acceleration due to gravity at P on the surface is

$$G = \frac{GM}{R^2} \qquad -(1)$$

Let the body be placed at Q at a height h from the surface of the Earth. The acceleration due to gravity at Q is

$$g_h = \frac{GM}{(R+h)^2} \qquad -(2)$$

Dividing (2) by (1) $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$



Variation of g with attitude

By simplifying and expanding using binomial theorem

$$g_h = g\left[1 - \frac{2h}{R}\right]$$

The value of acceleration due to gravity decreases with increase in height above the surface of the Earth.

(ii) Variation of g with depth

Consider the Earth to be a homogeneous sphere with uniform density of radius R and mass M.

Let P be a point on the surface of the Earth and Q be a point at a depth d from the surface.

The acceleration due to gravity at P on the surface is $g = GM/R^2$

If ρ be the density, then, the mass of the Earth is $M = 4/3\ \pi R^3 \rho$

$$g = 4/3\ G\pi R\rho \qquad -(1)$$

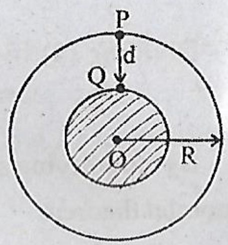
The acceleration due to gravity at Q at a depth d from the surface of the earth is

$$g_d = \frac{GM_d^{\ 2}}{(R-d)^2}$$

Where M_d is the mass of the inner sphere of the earth of radius (R – d)

$$M_d = 4/3\ \pi(R-d)^3\rho$$

$$g_d = 4/3\ G\pi(R-d)\rho \qquad -(2)$$



Variation of g with depth

dividing (2) by (1)

$$\frac{g_d}{g} = \frac{R-d}{R}$$

$$g_d = g\left[1 - \frac{d}{R}\right]$$

The value of acceleration due to gravity decreases with increase of depth.

Inertial mass

According to Newton's second law of motion (F = ma), the mass of a body can be determined by measuring the acceleration

produced in it by a constant force, (t.e) m = F/a. Internal mass of a body is a measure of the ability of a body to oppose the production of acceleration in it by an external force.

If a constant force acts on two masses m_A and m_B and produces accelerations aA and Og respectively, then, $F = m_A a_A = m_B a_B$

$$m_A/m_B = a_B/a_A$$

The ratio of two masses is independent of the constant force if die same force is applied on two different bodies, the inertial mass of the body is more in which the acceleration produced is less.

If one of the two masses is a standard kilogram, the unknown mass can be determined by comparing their accelerations.

Gravitational mass

According to Newton's law of gravitation, the gravitational force on a body is proportional to its mass. We can measure the mass of a body by measuring the gravitational force exerted on it by a massive body like Earth. Gravitational mass is the mass of a body which determines the magnitude of gravitational pull between the body and the Earth. This is determined with the help of a beam balance.

If F_A and F_B are the gravitational forces of attraction on the two bodies of masses m_A and m_B due to the Earth, then

$$F_A = \frac{Gm_A M}{R^2} \text{ and } F_B = \frac{Gm_B M}{R^2}$$

where M is mass of the Earth, R is the radius of the Earth and G is the gravitational constant.

$$m_A/m_B = F_A/F_B$$

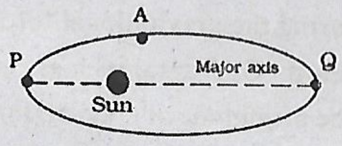
If one of the two masses is a standard kilogram, the unknown mass can be determined by comparing the gravitational forces.

Kepler's laws of planetary motion

i. The law of orbits

Each planet moves in an elliptical orbit with the Sun at one focus.

A is a planet revolving round the Sun. The position P of the planet where it is very close to the Sun is known as perigee and the position Q of the planet where it is farthest from the Sun is known as apogee.

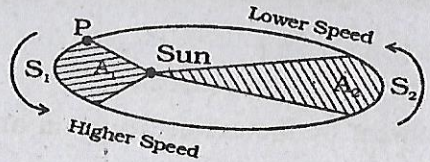


Law of orbits

ii. The law of areas

The line joining the Sun and the planet (i.e radius vector) sweeps out equal areas in equal interval of times.

The orbit of the planet around the Sun is as shown in figure. The areas A_1 and A_2 are swept by the radius vector in equal times. The planet covers unequal distances S_1 and S_2 in equal time. This is due to the variable speed of the planet. When the planet is closest to the Sun, it covers greater distance in a given time. Hence, the speed is maximum at the closest position. When the planet is far away from the Sun, it covers lesser distance in the same time. Hence the speed is minimum at the farthest position.

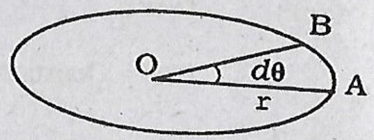


Law of areas

Proof for the law of areas

Consider a planet moving from A to B. The radius vector OA sweeps a small angle dθ at the centre in a small interval of time dt. From the figure, AB = rdθ. The small area dA swept by the radius is,

$$dA = \frac{1}{2} \times r \times r d\theta$$



Proof for the law of areas

Dividing by dt on both sides

$$\frac{dA}{dt} = \frac{1}{2} \times r^2 \times \frac{d\theta}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega$$

Where ω is the angular velocity.

The angular momentum is given by $L = mr^2\omega$

$$r^2\omega = \frac{L}{m}$$

Hence, $$\frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$$

Since the line of action of gravitational force passes through the axis, the external torque is zero. Hence, the angular momentum is conserved.

$$dA/dt = \text{constant}$$

(ie) the area swept by the radius vector in unit time is the same.

iii. The law of periods

The square of the period of revolution of a planet around the Sun is directly proportional to the cube of the mean distance between the planet and the Sun.

(i.e) $T^2 \alpha\ r^3$

$$T^2/r^3 = \text{Constant}$$

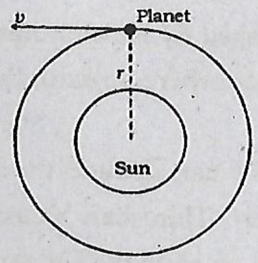
Proof for the law of periods

Let us consider a planet P of mass m moving with the velocity v around the Sun of mass M in a circular orbit of radius r.

The gravitational force of attraction of the Sun on the planet is.

$$F = \frac{GMm}{r^2}$$

The centripetal force is, $F = \frac{mv^2}{r}$



Proof for the law of periods

Equating the two forces

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{Gm}{r} \qquad - (1)$$

If T be the period of revolution of the planet around the Sun, then

$$v = \frac{2\pi r}{T} \qquad - (2)$$

Substituting (2) in (1) $\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

GM is a constant for any planet.

$$T^2 \alpha\ r^3$$

Newton's Formula of Universal Gravitation is Just Kepler's Third Law

Introduction

Stepwise analysis reveals that Newton's Formula of Universal Gravitation is just Kepler's Third Law. Newton fully plagiarized Kepler's formula, but added new definitions which were entirely groundless.

The Universal Gravitation Formula is commonly believed to be developed by Newton himself. But in his works, Newton didn't mention where on earth this formula was from.

We can deduce the Universal Gravitation Formula from Kepler's Third Law by using Newton's thoughts, and thus clarify where on earth the Universal Gravitation Formula was from.

Deduction

Kepler's Third Law is expressed as:

$$R^3/T^2 = K \qquad -(1)$$

R is the average orbital radius, and T is the orbiting period.

Because $T = 2\pi R/T$, substitute it into Formula (1), then

$$V^2R = 4\pi^2K \qquad -(2)$$

Put the variables at the left side of the formula and the constants at the right side.

Formula (2) would have immediately reminded Newton of the centripetal acceleration formula : $V^2/R = a$

Then transform the left side of Formula (2) to V^2/R and divide both sides by R^2,

Then :

$$\frac{V^2}{R} = \frac{4\pi^2K}{R^2} \qquad -(3)$$

The core of Newton's mechanics is $F = ma$, so he would change the left side of Formula (3) into F, namely $V^2/R\ .m$

Multiply both sides of Formula (3) by m (e.g. the mass of a planet in the solar system), then:

$$m\ .\ \frac{V^2}{R} = \frac{4\pi^2K.m}{R^2} \qquad -(4)$$

The next step is the most miraculous and important. When Newton saw "$4\pi^2K$" in Formula (4), he though it was too large. Time before Newton's era, people already knew k is a constant that is only decided by the central celestial body and independent from the orbiting celestial bodies. People also roughly knew a larger central celestial body has a larger k, and they might be in direct proportion. Following this clue, Newton made a very bold hypothesis or guess that "$4\pi^2K$" is the mass of a central celestial body. Then he found "$4\pi^2K$" and mass have different units, so accordingly, he introduced a "constant with unit", which is the gravitational constant G. At this moment, Newton provided at own willingness that: $MG = 4\pi^2K$, where M is the mass of the central celestial body. Substitute it into Formula (4), then

$$m\ .\ \frac{V^2}{R} = G\ \frac{M.m}{R^2} \qquad -(5)$$

$$F = ma = m.\frac{V^2}{R} = \frac{GM.m}{R^2}$$

Formula (5) is exactly the Universal Gravitation Formula that we are familiar with.

We now summarize the whole process. It is a regular transformation from Formula (1) (Kepler's Third Law) to Formula (4). But From Formula (4) to Formula (5), why did he replace "$4\pi^2K$" by MG, Newton didn't provide any reliable or verifiable proof.

In other words, someone else might provide "$4\pi^2K$" as the product of total sun energy and a constant: $NG = 4\pi^2K$.

G is the constant that balances units, N is total sun energy; substitute into Formula (4), then a "new universal gravitation formula" comes:

$$m.\frac{V^2}{R} = G\frac{N.m}{R^2}$$

$$F = ma = m.\frac{V^2}{R} = \frac{N.m}{R^2} \quad - (6)$$

If we calculate "the orbiting period, linear velocity, distance of celestial bodies, centripetal acceleration" of moving celestials with Formula (6), we can also get the results that are exactly the same as observations. Why? Because this formula is exactly Kepler's Third Law. "$4\pi^2K$" can be replaced by any "letter" without changing the essence of Kepler's Third Law. In this way, Newton replaced "ma" by the letter "F". With this "letter replacement", we can create

countless forms of Kepler's Third Law. This is just like calling a person by a hundred names with different letters, but he is still the same person.

Both Formula (5) (Newton's Formula of Universal Gravitation) and Formula (6) (new universal gravitation formula) are essentially Kepler's Third Law, because they only undergo some transformation and replacement. Therefore, Newton's Formula of Universal Gravitation and Kepler's Third Law are the same. By means of camouflaging, Newton transformed Kepler's Third Law, which is originally simple and pure in, into a gravitation formula, which is complicated and obscure. All achievements in astronomy and astronautics made by Newton's Formula of Universal Gravitation should be attributed to Kepler's Third Law. It is inappropriate to regard Universal Gravitation Formula and Kepler's Third Law as two formulae. When people are calculating celestial bodies and satellites with Universal Gravitation Formula, they don't realize they are using Kepler's Third Law.

Definition of G

If $m_1 = m_2 = 1$ kg and r = lm, then F = G. Thus the gravitational constant is equal to the force of attraction between two unit masses of matter, unit distance apart.

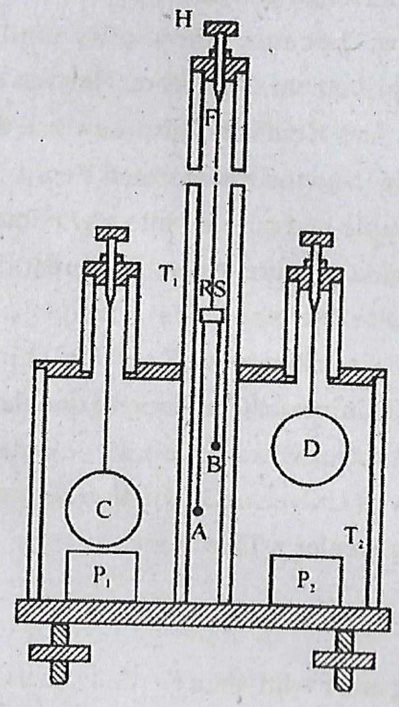
Determination of G

$$G = \frac{Fr^2}{m_1 m_2}$$ Dimensions of G are given by

$$[G] = \frac{MLT^{-2} \times L^2}{M^2} = M^{-1}L^3T^{-2}$$

In S.I. units. $G = 6.670 \times 10^{-11} Nm^2kg^{-2}$

Determination of G-Boy's Experiment



The apparatus consists of two co-axial glass tubes T_1 and T_2 mounted on a platform provided with levelling screws (figure). The inner tube T_1 is fixed, while the outer tube T_2 can be rotated about the common axis A small mirror, RS, is suspended in the inner tube by a fine quartz fibre F from a torsion head H. From the two ends of the mirror, two gold spheres A and B are suspended, such that the spheres are at different depths below the mirror. In the outer

co -axial tube T_2, two large lead balls C and D are suspended from its revolving lid such that the centre of C is in level with that of A, the centre of D is in level with that of B, and the distance AC is equal to BD. Two rubber pads P_1 and P_2 are placed below the two lead spheres, as a safeguard against damage, in case they should fall accidentally.

The experiment is performed by rotating the outer glass tube until the large lead spheres lie on the opposite sides of the two gold balls, so as to exert the maximum moment on the suspended system. In this position, the angle through which the mirror (RS) turns is maximum. The outer glass tube is then rotated, so that the lead spheres now lie on the other sides of the gold balls, in an exactly similar position producing the greatest deflection, the mean of these two observations gives the deflection of the mirror scale arrangement is used to measure θ.

Force of attraction between

$$\text{Spheres A and C} = \frac{GMm}{(AC)^2}$$

Force of attraction between

$$\text{Spheres B and D} = \frac{GMm}{(BD)^2}$$

Since AC = BD, the two forces are equal, parallel and act in opposite directions, thus constituting a couple.

The moment of the reflection couple

$$= \frac{GMm}{(AC)^2} \times 2l = \frac{GMm}{d^2} \times 2;$$

Where $2l$ = the length of the mirror strip RS and AC = d.

The deflection of the mirror strip under this couple is resisted the torsion or twist set up in the suspension fibre. The mirror strip comes to rest when the deflecting couple due to gravitational pull is balanced by the restoring torsional couple set up in the suspension fiber.

Now, if c be the torsional couple per unit twist, then for angular deflection 0, the total restoring couple is cθ.

In equilibrium position, $\frac{GMm}{d^2} \times 2l = c\theta$

From this, the value of G can be calculated. Using the arrangement of the quartz fibre and the mirror strip with gold balls as a torsion pendulum, the period T is found.

Then $T = 2\pi \frac{I}{C}$

where I = moment of inertia of the suspended system. From this, c can be calculated,

$$c = \frac{4\pi^2}{T} I$$

The results obtained by him are very accurate. The value obtained for G by Boys is 6.6576×10^{-11} Nm^2 Kg^{-2}.

Advantages

1. The size of the apparatus is very much reduced. The disturbances due to convection currents are therefore almost negligible.
2. By arranging the masses at different levels, the effect of the attraction of the heavier mass on the remote smaller mass is very much reduced.
3. By the lamp and scale arrangement, very small deflections can be measured accurately.
4. The use of a quartz fibre has made the apparatus very sensitive and accurate.

University Questions

2 Mark questions

1. State Newton's law of gravitation.
2. Define Gravitational constant.
3. Write advantages of G-Boy's experiment.
4. What are the special features of the gravitational law?
5. What is gravitational mass?